MA829 **PARTIAL DIFFERENTIAL EQUATIONS** **4-0-0-4**

Objective:

- To provide a platform for Ph. D students in first and second order partial differential equation.
- To demonstrate the suitability of different practical applications of PDE's and fourier transforms in solving wave equation, boundary conditions etc.

Keywords:

Multivariable Calculus, Linear and Quasilinear first-order PDEs, Cauchy Kowalevski Theorem, D'Alembert solution, Dirichlet problem, Heat flow problem, Green's Functions, Heat and Wave equations.

Pre-requisites:

1. Basic Multivariable
2. Calculus and ODEs

Content:

First order linear and quasi-linear PDEs, The Cauchy problem,Second order PDEs, Classification of PDEs, Characteristics, Well-posed problems, Fourier Series,Solutions of hyperbolic, parabolic and elliptic equations, Dirichlet and Neumann problems, Maximum principles, Fourier Transform methods for PDEs, The method of Green's functions for Laplace, Heat and wave equations.

Course Detail:

Sl. No.	Module/ Lecture Topics	No. of (Total) Hours
1	Mathematical Preliminaries: A Review of Multivariable Calculus,Essential Ordinary Differential Equations, Integral Curves and Surfaces of Vector Fields,Solving Equations of the form: dx/P=dy/Q=dz/R.	4
2	First-Order Partial Differential Equations(PDEs)– Formation and classification of first-order PDEs, Linear and Quasilinear first-order PDEs, Cauchy's problem for first order PDEs, The Cauchy Kowalevski Theorem, Integral surfaces passing through a given curve, Nonlinear first-order PDEs, The method of characteristics, Compatible systems, Charpit's method, Jacobi's method for nonlinear PDEs.	7
3	Second-Order PDEs - Classification, Canonical forms, Well-posed problems,Superposition principle.	3
4	Fourier Series (FS) – Introduction to FS, Convergence of FS for continuous and piecewise continuous functions, Differentiation and integration of FS, Fourier cosine and sine series.	3
5	The Heat Equation - Derivation of the heat equation, The maximum and	5

	minimum principles, Uniqueness, Continuous dependence, Method of separation of variables, Timeindependent boundary conditions, Time-dependent boundary conditions, Duhamel's principle.	
6	The Wave Equation - Derivation of the wave equation, The infinite string problem, The D'Alembert solution of the wave equation, The semi-infinite string problem, The finite vibrating string problem, The method of separation variables, The inhomogeneous wave equation.	5
7	Laplace's Equation – Basic concepts, Types of boundary value problems, The maximum and minimum principle, Green's identity and fundamental solution, The Poisson integral formula, The method of separation of variables, The Dirichlet problem for the rectangle, The Dirichlet problem for Annuli and Disk, The exterior Dirichlet problem.	6
8	The Fourier Transform Methods for PDEs –Fourier transform, Fourier sine and cosine transform, Heat flow problem in an infinite and semi-infinite rod, Infinite string problemand Laplace equation in a half-plane.	5
9	The Method of Green's Functions – Integral formulation, The method of Green's functions for the Laplace, Heat and Wave equations.	3
	Total	41

TEXT BOOKS / REFERENCES:

1. I.N. Sneddon, Elements of partial differential equations, Dover, 2013.
2. Claes Johnson, Numerical Solution of Partial Differential Equations by the Finite Element, Dovar, 2009.
3. *K. Deb,* "Optimization for Engineering Design – Algorithms and Examples", *Printice Hall, 1995.*
4. *S. S. Rao,* Engineering Optimization Theory and Practices", *New Age International(P) Ltd., 2000*.
5. D. Bleecker and G. Csordas, Basic Partial Differential Equations, Van Nostrand Reinhold, New York, 1992.
6. C. Constanda, Solution Techniques for Elementary Partial Differential Equations, Chapman & Hall/CRC, New York, 2002.
7. L. J. Crowin and R. H. Szczarba, Multivariable Calculus, Marcel Dekker, Inc, New York, 1982.
8. S. J. Farlow, Partial Differential Equations for Scientists and Engineers, Birkhauser, New York, 1993.

Outcome:

- Students will be able to formulate the wave equations, heat transfer equation, etc.
- Students will be able to choose the best optimization techniques while doing mathematical analysis.