

## Introduction

This intense targeted course designed for research (PhD) students working in the area of multi-body dynamics aims to develop students skill in solving advanced problems in mechanics, problems involving generalized coordinates and constraints using variational methods and introduce them to methods of numerically integrate differential equations of the equations of motion of point particles and rotating rigid bodies and train students in the technology of simulating such mechanical systems in MATLAB or any other computing framework.

## Course Outcomes

After completion of this course, students should be able to

CO1: demonstrate skill in using basic principles of mechanics to analyse and solve problems of moderate difficulty, including planetary motion, damped and driven harmonic oscillations, with application to vibration isolation, evaluate and interpret solutions.

CO2: Analyse and formulate dynamical systems using the concepts of generalized coordinates, momenta, variational principles, Lagrangian and Hamiltonian formalisms of mechanics, and derive Lagrangians and Hamiltonians for more difficult problems and those involving constrained motion, recognize symmetries and deduce associated conservation laws, and use them to simplify and solve the equations of motion, interpret solutions.

CO3: Analyse the kinematics and dynamics in rotating coordinate systems and concepts involved rigid body dynamics, including a representation of rotations using Euler angles and quaternion parameters, and solve associated equations of motion, describe and analyse the motion of spinning and precessing top.

CO4: study, evaluate and determine the motion of dynamical systems using various methods of numerically integrating ordinary differential equations of point particles, rigid bodies, systems with constraints, systems of first order ordinary differential equations in time, with adaptive step size control, conservation of energy, momentum and angular momentum, norm, and orthogonality methods to limit errors, evaluate the of stability some methods, and simulate motion using MATLAB or an equivalent tool.

## Syllabus

### UNIT 1

**Dynamic of Particles and System of Particles** (Refs 1–3): Review of concepts in motion of points particles and system of particles – equations of motion in cartesian and polar coordinates, work, kinetic and potential energies, conservative forces, angular momentum and torque, accelerating frames, friction. (CO1)

**Constrained Motion, Lagrangian and Hamiltonian Formalisms:** generalized coordinates, constraints – holonomic, non-holonomic, and other types, integrability, virtual displacements, virtual work, principle of virtual work, kinetic and potential energies, generalized momentum; impulse response.

Lagrangian and Hamilton's equations: D'Alembert's Principle & Lagrange's Equations, Hamilton's Equations, Integrals of Motion, Dissipative forces, Configuration Space & Phase-

Space, Impulse Response, application to simple mechanical problems with and without constraints. (CO2)

## **UNIT 2**

### **Rotating Reference Frames, Kinematics and Dynamics of a Rigid Body** (Refs 1–3):

Transformation of coordinates, rotating coordinate systems, motion in linearly accelerating and rotating frames, motion relative to rotating Earth; Kinematics of a rigid body, linear and angular momentum, translation theorem, kinetic energy, principal axis, equations of motion of a rigid body, Euler's equation of motion, Euler's angles, moment-free symmetric body, motion of a symmetric top. (CO3)

## **UNIT 3**

### **Solving Equations of Motion by Numerical Methods:** (Ref 1, Ch. 6, Ref 4., Ch. 13 & 14, Ref. 9)

Review of Basic Numerical Techniques: Errors in numerical computation, computational time, Taylor series methods and approximate of behaviour near points of interest, and numerical estimates, basics of numerical differentiation, integration, root finding and stability, interpolation, solving linear systems of equations, eigen values and vectors, using known routines or MATLAB; Plotting curves and surfaces in 2D and 3D.

Integrating equations of motion: state-vector and its evolution; explicit, implicit and improved Euler methods, predictor-corrector methods, Runge-Kutta methods and adaptive step size, extrapolative and linear multi-step methods, Verlet methods; Rotational Motion: body-coordinate system, properties of rotation and angular velocity matrices, momentum, angular momentum, equations of motion in matrix form; explicit and implicit methods and the loss of orthogonality; parametrization by Euler-angles and Cayley-Klein (quaternion) parameters. Numerical stability analysis for some methods; kinematic constraints, energy and momentum methods. Projects: MATLAB simulation of planetary orbits, simulation of free rotor and symmetric rotating top, collision contact of a falling ball with ground involving deformation of ground and dissipation. (CO4)

### **Text and References:**

1. Donald Greenwood, Classical Dynamics, Dover Publications, 1977.
2. Farid Amirouche, Fundamentals of Multibody Dynamics: Theory and Applications, Birkhauser (Ch. 1 & 2).
3. L. Meirovitch, Fundamentals of Analytical Mechanics, Dover Publications. (Ch. 1 – 4).
4. Philipp Scherer, Computational Physics Simulation of Classical and Quantum Systems, 3<sup>rd</sup> Ed., Chap13, 14.
5. N.C. Rana & P.S. Joag, Classical Mechanics, McGraw-Hill India.
6. Goldstein, Poole & Safko, Classical Mechanics, 3rd Ed, Pearson India.
7. J. Marion and S. Thornton, Classical Dynamics of Particles and Systems, Pearson India.
8. Kleppner and Kolenkov, Introduction to Mechanics, 1<sup>st</sup> Ed., McGraw-Hill India.
9. Erwin Kreyszig, Advanced Engineering Mathematics, 10E

### **Course Evaluation Policy:**

May vary from instructor to instructor for Ph.D. level courses.